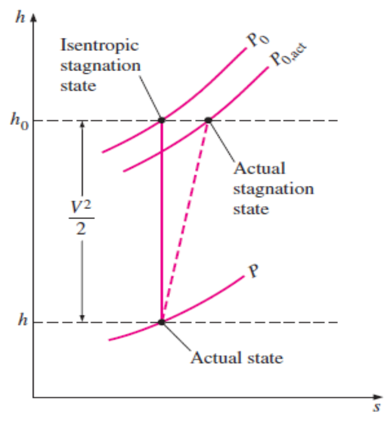
$$h_0 = h + \frac{V^2}{2} \qquad (\text{kJ/kg})$$

stagnation enthalpy

No work, no change in elevation →

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

انتالپی سکون نشان دهنده آنتالپی سیال است اگر به صورت آدیاباتیک به سرعت صفر رسانده شود.



$$c_p T_0 = c_p T + \frac{V^2}{2}$$

دمای سکون

$$T_0 = T + \frac{V^2}{2c_p}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = c_p(T_{02} - T_{01}) + g(z_2 - z_1)$$

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

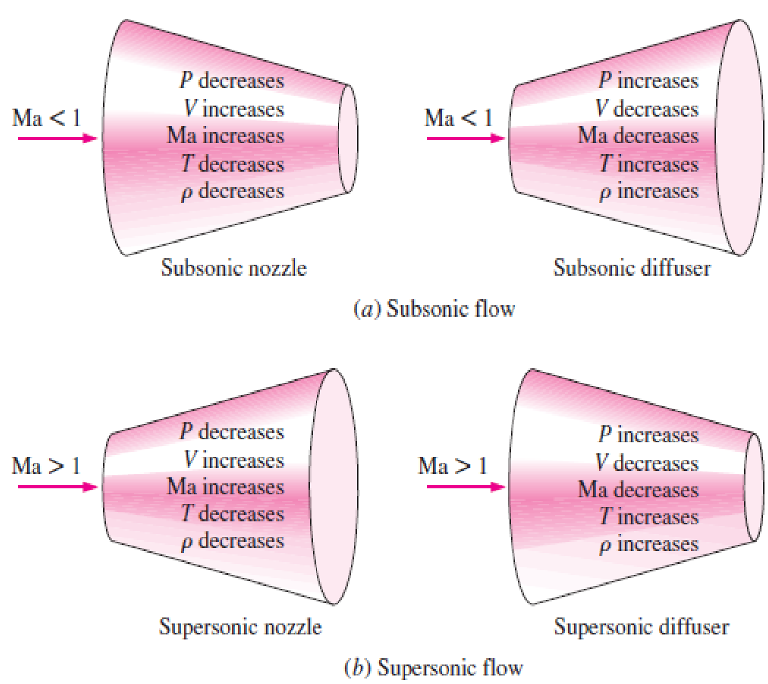
$$\dot{m} = \rho A V = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\frac{dP}{\rho} + V\,dV = 0$$

$$\frac{dA}{A} = \frac{dP}{\rho}\left(\frac{1}{V^2} - \frac{d\rho}{dP}\right)$$

$$\frac{dA}{A} = \frac{dP}{\rho V^2}(1 - \text{Ma}^2)$$



$$T_0 = T + \frac{V^2}{2c_p}$$

or

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT}$$

Noting that $c_p = kR/(k-1)$, $c^2 = kRT$, and $\mathrm{Ma} = V/c$, we see that

$$\frac{V^2}{2c_pT} = \frac{V^2}{2[kR/(k-1)]T} = \left(\frac{k-1}{2}\right)\frac{V^2}{c^2} = \left(\frac{k-1}{2}\right)\mathrm{Ma}^2$$

Substituting yields

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{k/(k-1)} \qquad \text{(17–19)}$$

stagnation to static density is obtained by substituting
ı. 17–6:

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{1/(k-1)} \qquad \text{(17–20)}$$

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

یک نازل همگرا می تواند به عنوان یک کنترل گر دبی جریان در جریان های سونیک باشد.(A*)

$$\dot{m}_{\max} = A^*P_0\sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

$$\dot{m} = \rho AV = \left(\frac{P}{RT}\right)A\left(\mathrm{Ma}\sqrt{kRT}\right) = PA\mathrm{Ma}\sqrt{\frac{k}{RT}}$$

Solving for T from Eq. 17–18 and for P from Eq. 17–19 and substituting,

$$\dot{m} = \frac{A\mathrm{Ma}P_0\sqrt{k/(RT_0)}}{[1 + (k-1)\mathrm{Ma}^2/2]^{(k+1)/[2(k-1)]}} \qquad \text{(17–24)}$$

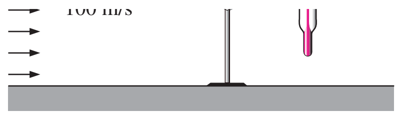


FIGURE 17–5

The temperature of an ideal gas flowing at a velocity V rises by $V^2/2c_p$ when it is brought to a complete stop.

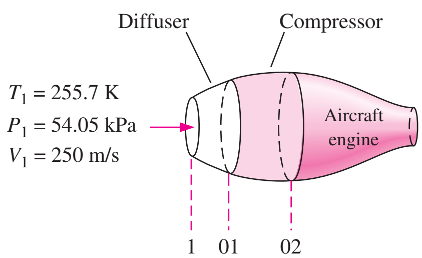


FIGURE 17–6

Schematic for Example 17–1.

EXAMPLE 17–1 Compression of High-Speed Air in an Aircraft

An aircraft is flying at a cruising speed of 250 m/s at an altitude of 5000 m where the atmospheric pressure is 54.05 kPa and the ambient air temperature is 255.7 K. The ambient air is first decelerated in a diffuser before it enters the compressor (Fig. 17–6). Assuming both the diffuser and the compressor to be isentropic, determine (*a*) the stagnation pressure at the compressor inlet and (*b*) the required compressor work per unit mass if the stagnation pressure ratio of the compressor is 8.

Solution High-speed air enters the diffuser and the compressor of an aircraft. The stagnation pressure of air and the compressor work input are to be determined.

Assumptions **1** Both the diffuser and the compressor are isentropic. **2** Air is an ideal gas with constant specific heats at room temperature.

Properties The constant-pressure specific heat c_p and the specific heat ratio k of air at room temperature are (Table A–2a)

$$c_p = 1.005 \text{ kJ/kg}\cdot\text{K} \quad \text{and} \quad k = 1.4$$

Analysis (*a*) Under isentropic conditions, the stagnation pressure at the compressor inlet (diffuser exit) can be determined from Eq. 17–5. However, first we need to find the stagnation temperature T_{01} at the compressor inlet. Under the stated assumptions, T_{01} can be determined from Eq. 17–4 to be

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 255.7 \text{ K} + \frac{(250 \text{ m/s})^2}{(2)(1.005 \text{ kJ/kg}\cdot\text{K})}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$
$$= 286.8 \text{ K}$$

Then from Eq. 17–5,

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (54.05 \text{ kPa})\left(\frac{286.8 \text{ K}}{255.7 \text{ K}}\right)^{1.4/(1.4-1)}$$
$$= \mathbf{80.77\ kPa}$$

That is, the temperature of air would increase by 31.1°C and the pressure by 26.72 kPa as air is decelerated from 250 m/s to zero velocity. These increases in the temperature and pressure of air are due to the conversion of the kinetic energy into enthalpy.

(*b*) To determine the compressor work, we need to know the stagnation temperature of air at the compressor exit T_{02}. The stagnation pressure ratio across the compressor P_{02}/P_{01} is specified to be 8. Since the compression process is assumed to be isentropic, T_{02} can be determined from the ideal-gas isentropic relation (Eq. 17–5):

$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (286.8 \text{ K})(8)^{(1.4-1)/1.4} = 519.5 \text{ K}$$

Disregarding potential energy changes and heat transfer, the compressor work per unit mass of air is determined from Eq. 17–8:

$$w_{in} = c_p(T_{02} - T_{01})$$
$$= (1.005 \text{ kJ/kg}\cdot\text{K})(519.5 \text{ K} - 286.8 \text{ K})$$
$$= \mathbf{233.9 \text{ kJ/kg}}$$

Thus the work supplied to the compressor is 233.9 kJ/kg.

Discussion Notice that using stagnation properties automatically accounts for any changes in the kinetic energy of a fluid stream.

EXAMPLE 17–3 Gas Flow through a Converging–Diverging Duct

Carbon dioxide flows steadily through a varying cross-sectional-area duct such as a nozzle shown in Fig. 17–12 at a mass flow rate of 3 kg/s. The carbon dioxide enters the duct at a pressure of 1400 kPa and 200°C with a low velocity, and it expands in the nozzle to a pressure of 200 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to a pressure drop of 200 kPa.

Solution Carbon dioxide enters a varying cross-sectional-area duct at specified conditions. The flow properties are to be determined along the duct.

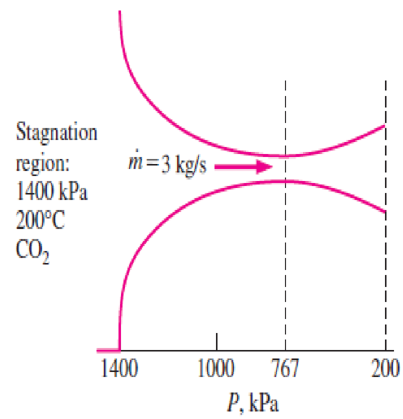


FIGURE 17–12

Schematic for Example 17–3.

Assumptions **1** Carbon dioxide is an ideal gas with constant specific heats at room temperature. **2** Flow through the duct is steady, one-dimensional, and isentropic.

Properties For simplicity we use c_p = 0.846 kJ/kg · K and k = 1.289 throughout the calculations, which are the constant-pressure specific heat and specific heat ratio values of carbon dioxide at room temperature. The gas constant of carbon dioxide is R = 0.1889 kJ/kg · K (Table A–2a).

Analysis We note that the inlet temperature is nearly equal to the stagnation temperature since the inlet velocity is small. The flow is isentropic, and thus the stagnation temperature and pressure throughout the duct remain constant. Therefore,

$$T_0 \cong T_1 = 200°\text{C} = 473 \text{ K}$$

and

$$P_0 \cong P_1 = 1400 \text{ kPa}$$

To illustrate the solution procedure, we calculate the desired properties at the location where the pressure is 1200 kPa, the first location that corresponds to a pressure drop of 200 kPa.

From Eq. 17–5,

$$T = T_0\left(\frac{P}{P_0}\right)^{(k-1)/k} = (473 \text{ K})\left(\frac{1200 \text{ kPa}}{1400 \text{ kPa}}\right)^{(1.289-1)/1.289} = 457 \text{ K}$$

From Eq. 17–4,

$$\begin{aligned} V &= \sqrt{2c_p(T_0 - T)} \\ &= \sqrt{2(0.846 \text{ kJ/kg}\cdot\text{K})(473 \text{ K} - 457 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^3}{1 \text{ kJ/kg}}\right)} \\ &= \mathbf{164.5 \text{ m/s}} \end{aligned}$$

From the ideal-gas relation,

$$\rho = \frac{P}{RT} = \frac{1200 \text{ kPa}}{(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(457 \text{ K})} = \mathbf{13.9 \text{ kg/m}^3}$$

From the mass flow rate relation,

$$A = \frac{\dot{m}}{\rho V} = \frac{3 \text{ kg/s}}{(13.9 \text{ kg/m}^3)(164.5 \text{ m/s})} = 13.1 \times 10^{-4} \text{ m}^2 = \mathbf{13.1 \text{ cm}^2}$$

From Eqs. 17–11 and 17–12,

$$c = \sqrt{kRT} = \sqrt{(1.289)(0.1889 \text{ kJ/kg}\cdot\text{K})(457 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 333.6 \text{ m/s}$$

$$\text{Ma} = \frac{V}{c} = \frac{164.5 \text{ m/s}}{333.6 \text{ m/s}} = \mathbf{0.493}$$

The results for the other pressure steps are summarized in Table 17–1 and are plotted in Fig. 17–13.

Discussion Note that as the pressure decreases, the temperature and speed of sound decrease while the fluid velocity and Mach number increase in the flow direction. The density decreases slowly at first and rapidly later as the fluid velocity increases.

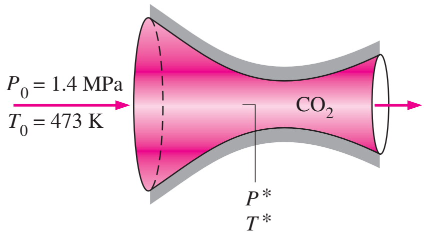


FIGURE 17–19

Schematic for Example 17–4.

EXAMPLE 17–4 Critical Temperature and Pressure in Gas Flow

Calculate the critical pressure and temperature of carbon dioxide for the flow conditions described in Example 17–3 (Fig. 17–19).

Solution For the flow discussed in Example 17–3, the critical pressure and temperature are to be calculated.

Assumptions **1** The flow is steady, adiabatic, and one-dimensional. **2** Carbon dioxide is an ideal gas with constant specific heats.

Properties The specific heat ratio of carbon dioxide at room temperature is $k = 1.289$ (Table A–2a).

Analysis The ratios of critical to stagnation temperature and pressure are determined to be

$$\frac{T^*}{T_0} = \frac{2}{k+1} = \frac{2}{1.289+1} = 0.8737$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = \left(\frac{2}{1.289+1}\right)^{1.289/(1.289-1)} = 0.5477$$

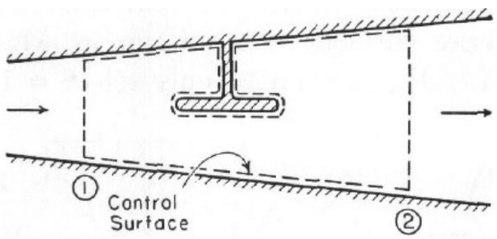
Noting that the stagnation temperature and pressure are, from Example 17–3, $T_0 = 473$ K and $P_0 = 1400$ kPa, we see that the critical temperature and pressure in this case are

$$T^* = 0.8737T_0 = (0.8737)(473\text{ K}) = \mathbf{413\text{ K}}$$

$$P^* = 0.5477P_0 = (0.5477)(1400\text{ kPa}) = \mathbf{767\text{ kPa}}$$

Discussion Note that these values agree with those listed in Table 17–1, as expected. Also, property values other than these at the throat would indicate that the flow is not critical, and the Mach number is not unity.

در جریان های تراکم پذیر عبارت $F=P.A+\rho AV^2$ زیاد دیده می شود. با نوشتن معادله بقای مومنتم برای یک مسیر جریان مانند شکل زیر نشان دهید این رابطه چگونه استفاده می شود. سپس آن را بر حسب خصوصیات جریان به دست آورده و بی بعد کنید. عبارتی برای F* پیشنهاد کنید.



$$\mathfrak{I} + p_1A_1 - p_2A_2 = \rho_2A_2V_2{}^2 - \rho_1A_1V_1{}^2$$

or

$$\mathfrak{I} = (p_2A_2 + \rho_2A_2V_2{}^2) - (p_1A_1 + \rho_1A_1V_1{}^2) = F_2 - F_1 \qquad (4.21)$$

$$\rho V^2 \equiv \frac{p}{RT}V^2 \equiv \frac{p}{kRT}kV^2 = kp\mathsf{M}^2$$

$$F = pA(1 + k\mathsf{M}^2)$$

$$\frac{F}{F^*} = \frac{p}{p^*}\cdot\frac{A}{A^*}\cdot\frac{1+k\mathsf{M}^2}{1+k} = \frac{p}{p_0}\cdot\frac{p_0}{p^*}\cdot\frac{A}{A^*}\cdot\frac{1+k\mathsf{M}^2}{1+k}$$

substituting p/p_0, p_0/p^*, and A/A^* from Eqs. 4.14b, 4.15b, and 4.19, respectively, there is obtained after simplification,

$$\frac{F}{F^*} = \frac{1+k\mathsf{M}^2}{\mathsf{M}\sqrt{2(k+1)\left(1+\frac{k-1}{2}\mathsf{M}^2\right)}} \qquad (4.24)$$

$$\frac{F}{p_0A^*} = \frac{p}{p_0}\frac{A}{A^*}(1 + k\mathsf{M}^2)$$

EXAMPLE 17–5 Effect of Back Pressure on Mass Flow Rate

Air at 1 MPa and 600°C enters a converging nozzle, shown in Fig. 17–24, with a velocity of 150 m/s. Determine the mass flow rate through the nozzle for a nozzle throat area of 50 cm^2 when the back pressure is (*a*) 0.7 MPa and (*b*) 0.4 MPa.

Solution Air enters a converging nozzle. The mass flow rate of air through the nozzle is to be determined for different back pressures.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The constant-pressure specific heat and the specific heat ratio of air are c_p = 1.005 kJ/kg · K and k = 1.4, respectively (Table A–2a).

Analysis We use the subscripts *i* and *t* to represent the properties at the nozzle inlet and the throat, respectively. The stagnation temperature and pressure at the nozzle inlet are determined from Eqs. 17–4 and 17–5:

$$T_{0i} = T_i + \frac{V_i^2}{2c_p} = 873\text{ K} + \frac{(150\text{ m/s})^2}{2(1.005\text{ kJ/kg}\cdot\text{K})}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 884\text{ K}$$

$$P_{0i} = P_i\left(\frac{T_{0i}}{T_i}\right)^{k/(k-1)} = (1\text{ MPa})\left(\frac{884\text{ K}}{873\text{ K}}\right)^{1.4/(1.4-1)} = 1.045\text{ MPa}$$

These stagnation temperature and pressure values remain constant throughout the nozzle since the flow is assumed to be isentropic. That is,

$$T_0 = T_{0i} = 884\text{ K} \quad \text{and} \quad P_0 = P_{0i} = 1.045\text{ MPa}$$

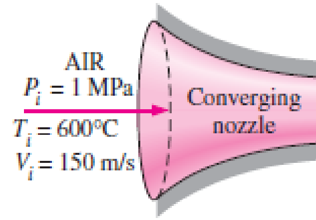


FIGURE 17–24
Schematic for Example 1

The critical-pressure ratio is determined from Table 17–2 (or Eq. 17–22) to be $P^*/P_0 = 0.5283$.

(*a*) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.7\ \text{MPa}}{1.045\ \text{MPa}} = 0.670$$

which is greater than the critical-pressure ratio, 0.5283. Thus the exit plane pressure (or throat pressure P_t) is equal to the back pressure in this case. That is, $P_t = P_b = 0.7$ MPa, and $P_t/P_0 = 0.670$. Therefore, the flow is not choked. From Table A–32 at $P_t/P_0 = 0.670$, we read $\text{Ma}_t = 0.778$ and $T_t/T_0 = 0.892$.

The mass flow rate through the nozzle can be calculated from Eq. 17–24. But it can also be determined in a step-by-step manner as follows:

$$T_t = 0.892T_0 = 0.892(884\ \text{K}) = 788.5\ \text{K}$$

$$\rho_t = \frac{P_t}{RT_t} = \frac{700\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(788.5\ \text{K})} = 3.093\ \text{kg/m}^3$$

$$V_t = \text{Ma}_t c_t = \text{Ma}_t\sqrt{kRT_t}$$

$$= (0.778)\sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(788.5\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)}$$

$$= 437.9\ \text{m/s}$$

Thus,

$$\dot{m} = \rho_t A_t V_t = (3.093\ \text{kg/m}^3)(50 \times 10^{-4}\ \text{m}^2)(437.9\ \text{m/s}) = \mathbf{6.77\ kg/s}$$

(*b*) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.4\ \text{MPa}}{1.045\ \text{MPa}} = 0.383$$

which is less than the critical-pressure ratio, 0.5283. Therefore, sonic conditions exist at the exit plane (throat) of the nozzle, and Ma = 1. The flow is choked in this case, and the mass flow rate through the nozzle can be calculated from Eq. 17–25:

$$\dot{m} = A^*P_0\sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

$$= (50 \times 10^{-4}\ \text{m}^2)(1045\ \text{kPa}) \times \sqrt{\frac{1.4}{(0.287\ \text{kJ/kg}\cdot\text{K})(884\ \text{K})}}\left(\frac{2}{1.4+1}\right)^{2.4/0.8}$$

$$= \mathbf{7.10\ kg/s}$$

since $\text{kPa}\cdot\text{m}^2/\sqrt{\text{kJ/kg}} = \sqrt{1000}\ \text{kg/s}$.

Discussion This is the maximum mass flow rate through the nozzle for the specified inlet conditions and nozzle throat area.